Lecture 3C
Dynamic Labor Supply

Labor Economics
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Overview

• Labor supply is clearly part of a lifetime decision making process.
  – attend school early in life
  – accumulate wealth while in labor force
  – make retirement decisions late in life

• Savings from labor earnings are often required to sustain living when out of the LF

• Variations in health, family composition and wages provide incentives for people to vary the timing of their labor earnings.
Some Stylized Facts

• The typical age-earnings profile has a predictable path
  (1) low when young
  (2) rise as worker ages, peak at about 50
  (3) remain stable or decline slightly after 50

• Hours worked generally rises with age, then fall before retirement, and goes to 0 at retirement.

• Hours of work and the wage rate move together over time for a particular worker.
The Life Cycle Path of Wages and Hours for a Typical Worker

(a) The age-earnings profile of a typical worker rises rapidly when the worker is young, reaches a peak at around age 50, and then wages either stop growing or decline slightly. (b) The changing price of leisure over the life cycle implies that the worker will devote relatively more hours to the labor market when the wage is high and fewer hours when the wage is low.
A Basic Framework

• Our utility function should be in a life cycle setting

\[ U = U(C_1, \ldots, C_t, \ldots, C_T; L_1, \ldots, L_t, \ldots, L_T) \]

• But we can obtain analytically simple and easily interpretable results from the function.

• So we have assume that it is temporally separable, thus

\[ U = \sum_{t=1}^{T} U(C_t, L_t, t) \]
A Basic Framework

• For a given initial value $A_0$, the wealth of the consumer is described by

$$A_t = (1 + r_t)A_{t-1} + B_t + w_t (1 - L_t) - C_t, \quad \forall t \geq 1$$

• It means that the wealth at any time $t$ will equal to three parts.
  (1) Savings and interests at $t$-1 time.
  (2) Non labor income at $t$ time.
  (3) Labor income at $t$ time.
A Basic Framework

• The optimal solutions

\[ \ell = \sum_{t=1}^{T} U(C_t, L_t, t) \]

\[ -\lambda_t \sum_{t=1}^{T} \left[ A_t - (1 + r_t) A_{t-1} - B_t - w_t (1 - L_t) + C_t \right] \]

• Then based on F.O.C, we can obtain

\[ U_C(C_t, L_t, t) = \lambda_t \]

\[ U_L(C_t, L_t, t) = \lambda_t w_t \]

\[ \lambda_t = (1 + r_{t+1}) \lambda_{t+1} \]
A Basic Framework

• The equality between the MRT and the current wage is maintained at every data.

• Then the optimal consumption are implicitly written

\[ C_t = C(w_t, \lambda_t, t) \]
\[ L_t = L(w_t, \lambda_t, t) \]

• The Frischian demands for period t
A Basic Framework

• Because

\[ \lambda_t = (1 + r_{t+1}) \lambda_{t+1} \]

• So

\[ \lambda_t = \left[ (1 + r_t)^{-1} \times (1 + r_{t-1})^{-1} \times \ldots \times (1 + r_1)^{-1} \right] \lambda_0 \]

• Then

\[ \ln \lambda_t = - \sum_{i=t}^{\infty} \ln(1 + r_i) + \ln \lambda_0 \]

• It can be breaks down into a fixed individual effect and an age effect common to all agents from empirical point of view.
Two Types of Wage Changes

1. Evolutionary wage change: A wage changes along with the worker’s wage-earning profile. So It has no impact whatsoever on the worker’s total lifetime income.

Two Types of Wage Changes

- Evolutionary Wage Change
- Parametric Wage Change
Two Predicted Wage Effects

• Evolutionary wage change implies that hours of work and the wage rate should move together over time.

• This implication differs from the conclusion in the static model which wage increase generate income and substitute effects.

• If income effect dominate, then there could be a negative relationship between wage and working hours.
Two Predicted Wage Effects

• Why? Because there is a huge different in “wage change” between two models.

• **Static model**: wage increase means that it expands the worker’s opportunity set and hence creates an income effect that increase the demand of leisure.

• **Lifecycle model**: the wage change has no impact whatsoever on the worker’s total lifetime income to a particular worker, and does not change the set.
Parametric Wage Change

• If we compare two works with different earnings profiles, then the difference in hours of work between them would be affected by both Substitution effects and income effects.

• Labor supply may be more or less depending on which effects are stronger.
Two Predicted Wage Effects

FIGURE 2-19  Hours of Work over the Life Cycle for Two Workers with Different Wage Paths

Joe’s wage exceeds Jack’s at every age. Although both Joe and Jack work more hours when the wage is high, Joe works more hours than Jack only if the substitution effect dominates. If the income effect dominates, Joe works fewer hours than Jack.
Wage and Labor Participation

• **Labor Participation depends on the reservation wage.**
• In each period of lifecycle, the worker will make decision.
• The person is then more likely to enter the labor market in periods when the wage is high.
• Then LPR are likely to be low for young workers, high for in their working years and low again for older workers.
FIGURE 2-20
Labor Force Participation Rates over the Life Cycle, 2005

FIGURE 2-21
Hours of Work over the Life Cycle, 2005

Estimation of Life Cycle Models

- The estimation of the intertemporal labor supply elasticity
- \( \Delta h_{it} = \sigma \Delta w_{it} + \text{other variables} \)
  - where \( \Delta H_{it} \) gives the year-to-year change in hours of work
  - \( \Delta w_{it} \) gives the year-to-year change in the worker’s wages.
  - The coefficient \( \sigma \) would be related to the intertemporal labor supply elasticity because it measures the change in hours of work for a given person resulting from a particular change in his wage rate.
Labor Supply Over the Business Cycle

• Added worker effect:
  – As the main breadwinner becomes unemployed, family income falls and the secondary workers get jobs to make up the loss
  – Labor force participation rate of secondary workers has a counter-cyclical trend

• Discouraged worker effect:
  – Many unemployed workers find it almost impossible to find jobs during a recession, and simply give up
  – The labor force participation rate has a pro-cyclical effect
Implications for Unemployment Rate

• The added worker effect increases the unemployment rate.
• The discouraged worker effect depresses the unemployment rate.
• Empirical evidence: The discourage worker effect is dominant.
• Official unemployment rate data may understate the unemployment problem during severe recession.
Application: Retirement Decisions

• It is not linked to the deteriorating health of this particular age group.
• Partially attribute to an increase in pension benefits.
• there is a strong link between the availability of private pension plans and the labor force participation of older men.
Social Security Disability Program

• Some studies instead argue that it can be attributed to the work disincentives created by the Social Security Disability Program.

• The monthly disability benefit equals the Social Security retirement benefits that the worker would have received had he or she continued working until age 65.

• Many workers would like to claim that they are disabled in order to enjoy the leisure activities associated with early retirement.
**Difference-in-Differences**

The difference-in-differences estimator is 2.7%.

It implies that the increased generosity of the disability program increased the proportion of men who did not work by 2.7 percentage points.
The Social Security Earnings Test

- Many workers who consider themselves retired continue to work, perhaps in a part-time job. In the United States, nearly 20% of “retired” persons also hold a job.

- Until 2000, the Social Security system had a provision, known as the Social Security earnings test, that presumably discouraged Social Security recipients from working.
  
  - Annual income < $17000, without affecting their SS benefit.

- If > $17000, the government reduced the size of the Social Security benefit. In particular, $1 of Social Security benefits was withheld for every $3 earned above the exempt amount, so that workers who earned more than $17,000 implicitly faced a 33 percent tax rate.
The Social Security Earnings Test
Labor Supply(IV)

Cross-Section Estimates of Labor Supply Elasticities
Overview

• Probably the greatest number of empirical studies area in labor economics.

• Beginning in 1970s, it accompanied with a lot of advanced econometric methods over the last twenty years.

• Now it seems to be a developed area in LE, but still active in some areas
Motivation

• Estimating a parameter which is mostly the size of the labor supply response to wages for policy purposes. (reduce form)

• Testing the predictions of the static labor supply model, such as estimating responses to $V$ and $w$ in the data to computer a pure substitution effect to see if it had the predicted sign. (structural)
Data and Setting

• Individual or family labor supply, with home production:
  – cross section

• Intertemporal labor supply:
  – panel data
Labor Supply Elasticity

- In the classical economics context, estimation the effect of price (wage) on supply (labor) is estimation the supply price elasticity.
- Thus labor supply elasticity is

\[
\gamma = \frac{\Delta h^* (w, V)}{\Delta w} \times 100\% \\
\gamma = \frac{\partial h^* (w, V)}{\partial w} \frac{w}{h^*} \quad \text{or} \quad \gamma = \frac{\partial \ln h^* (w, V)}{\partial \ln w}
\]
Labor Supply Elasticity

• Recall Slutsky equation

\[ \frac{\partial h}{\partial w} = \frac{\partial h^H}{\partial w} \bigg|_{U=U^*} + \frac{\partial h}{\partial Y} \chi h \]

• Multiplying the entire equation by \( w/h \)

\[ \frac{\partial h}{\partial w} \times \frac{w}{h} = \frac{\partial h^H}{\partial w} \bigg|_{U=U^*} \times \frac{w}{h} + \frac{\partial h}{\partial Y} \times \frac{Y}{h} \times \frac{wh}{Y} \]

• Thus

\[ \varepsilon_{hw} = \varepsilon_{hw}^c + \varepsilon_{hY} \times \frac{wh}{Y} \]
Labor Supply Elasticity

• Thus, there are three “sufficient statistics” of labour supply

(1) the uncompensated wage elasticity:
   the sign is ambiguous

(2) the compensated wage elasticity: the sign is positive - pure substitution effect

(3) the income elasticity: the sign is expected to be negative
Estimation in reality

- Holding An individual data set which contain some information of work for pay in certain period as followed

1. Total hours work
2. The hourly wage rate
3. Some measure of non-labor income including investment income, government transfer and spouse’s earnings.
4. A vectors of demographic characteristics, such as age, race, education, the number of kids etc.
Estimation in reality

• Typical regression function is followed

$$\ln h = a + b \ln w_i + c \ln V_i + dX_i + e_i$$

• $X$ is the control variables such as the presence of small children and tastes for work.

• So the estimate of $b$ is the elasticities of labor supply to wage and the estimate of $c$ is the elasticities of labor supply to nonwage income.
Some potential problems

- Omitted Variables Bias (OVB)
- Self-Selection Bias
- Measurement error
Omitted Variables Bias (OVB)

- OVB means when we estimate a regression, we omitted some important variables which can affect the outcomes \( y \) and explanatory variables \( x \).
- In labor supply case, there are some unobservable variables can affect both wage and working hours.
- What happen if the variable we want control is unobservable?
Omitted Variable Bias

Suppose the true model is given as
\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u, \] but we estimate
\[ \tilde{y} = \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + u, \] then

\[ \tilde{\beta}_1 = \frac{\sum (x_{i1} - \bar{x}_1) y_i}{\sum (x_{i1} - \bar{x}_1)^2} \]
Omitted Variable Bias (cont)

Recall the true model, so that

\[ y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + u_i \], so the numerator becomes

\[
\sum (x_{i1} - \bar{x}_1) (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + u_i) = \\
\beta_1 \sum (x_{i1} - \bar{x}_1)^2 + \beta_2 \sum (x_{i1} - \bar{x}_1) x_{i2} + \sum (x_{i1} - \bar{x}_1) u_i
\]
Omitted Variable Bias (cont)

\[ \tilde{\beta} = \beta_1 + \beta_2 \frac{\sum (x_{i1} - \bar{x}_1)x_{i2}}{\sum (x_{i1} - \bar{x}_1)^2} + \frac{\sum (x_{i1} - \bar{x}_1)u_i}{\sum (x_{i1} - \bar{x}_1)^2} \]

since \( E(u_i) = 0 \), taking expectations we have

\[ E(\tilde{\beta}_1) = \beta_1 + \beta_2 \frac{\sum (x_{i1} - \bar{x}_1)x_{i2}}{\sum (x_{i1} - \bar{x}_1)^2} \]
Omitted Variable Bias

So consider the regression $X_2$ on $X_1$, we get

$$\tilde{x}_2 = \delta_0 + \delta_1 x_1 \text{ then } \tilde{\delta}_1 = \frac{\sum (x_{i1} - \bar{x}_1)x_{i2}}{\sum ((x_{i1} - \bar{x}_1)^2)}$$

so $E(\tilde{\beta}_1) = \beta_1 + \beta_2 \tilde{\delta}_1 = \beta_1 + \beta_2 \frac{\text{cov}(x_1, x_2)}{\text{var}(x_1)}$
### Summary of Direction of Bias

<table>
<thead>
<tr>
<th>$\beta_2$</th>
<th>Corr($x_1$, $x_2$) &gt; 0</th>
<th>Corr($x_1$, $x_2$) &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2 &gt; 0$</td>
<td>Positive bias</td>
<td>Negative bias</td>
</tr>
<tr>
<td>$\beta_2 &lt; 0$</td>
<td>Negative bias</td>
<td>Positive bias</td>
</tr>
</tbody>
</table>
Omitted Variable Bias Summary

- Two cases where bias is equal to zero
  - $\beta_2 = 0$, that is $x_2$ doesn’t really belong in model
  - $x_1$ and $x_2$ are uncorrelated in the sample

- If correlation between $x_2$, $x_1$ and $x_2$, $y$ is the same direction, bias will be positive
- If correlation between $x_2$, $x_1$ and $x_2$, $y$ is the opposite direction, bias will be negative
There are other determinants of labor supply that could potentially be correlated with \( w \) and \( Y \), but we can’t put it into regression.

For example, a taste for work labeled by “ambition” means that people with high values of the taste tend to win promotions at faster rate than others.

Then they have both longer working hours and higher wages in cross section data.
OVB in Labor Supply Context

- Then $b$ in estimation in cross section regression is biased upwards.

- At the same time, such people will likely have higher nonwage incomes $Y$.

- Then $c$ in estimation (which is negative) in estimation biased upwards, i.e., towards zero.
How to Correct OVB?

- Experiment
- Instrumental variables (IV)
- Difference in Difference (DID)
Experiment

• Imbens, Rubin, and Sacerdote (2001) analyze the effects of the magnitude of lottery prizes on earnings, consumption and savings using an original survey of people playing the lottery in Massachusetts in the mid-1980s.