# Lec3: Labor Supply(II) 

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## Zhaopeng Qu

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Business School,Nanjing University


## Review

- Each of us in modern economic society have to make the following decisions

1. Whether to work
2. How many hours to work if employed

## Motivation

- Family is the basic unit in the modern economic world.
- Labor supply decision is made within the Family.
- Now a lot of economist think family or household economics is a independent sub-discipline in Economics.


## A Great Guy

- Professor at UChicago
- the 1992 Nobel Prize Laureate
- Pioneered study in the fields of human capital, economics of the family, and economic analysis of crime...


Gary Becker(1930-2014)

## Content

－More extension for labor supply model in the direction of family or household economics since 1960s．

1．Home production（家庭生产）by Gary Beck（1965）and Reuben Gronau（1977）．

2．Family labor supply（家庭内部决策）by Gary Beck（1983），Chiappori $(1988,1992)$ ．

Home Production

## Home Production

- Time devoted to household tasks is generally distinguished from leisure.
- We also can purchased them in the market.
- So individual's labor supply decisions also depends on effectiveness at performing household tasks versus doing paid work.


## Weekly Hours Spent in Household by Men and Women over Age 18, 2005 in USA

|  | Households with Children < 6 |  | Households with Children 6-I7 |  | Households with No Children < 18 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Women | Men | Women | Men | Women | Men |
| Paid Work ${ }^{\text {a }}$ | 19 | 39 | 25 | 39 | 22 | 29 |
| Household Work ${ }^{\text {b }}$ | 43 | 23 | 34 | 19 | 25 | 17 |
| Leisure ${ }^{\text {c }}$ | 29 | 32 | 33 | 35 | 42 | 45 |
| Personal Care ${ }^{\text {d }}$ | 74 | 72 | 74 | 72 | 76 | 74 |

${ }^{\text {a }}$ Includes commuting time
${ }^{\mathrm{b}}$ Includes time spent purchasing goods and services
${ }^{\text {c }}$ Includes time spent in volunteer activities
${ }^{\mathrm{d}}$ Includes time spent sleeping and eating
Source: U.S. Department of Labor, Bureau of Labor Statistics, "American Time Use Survey, 2005," Table 8 at http://www.bls.gov/news.release/atus.toc.htm.

## Purpose of Household Production

- Explain patterns of household time allocation.
- The allocation of time between house work, market work, and leisure
- The division of labor in a households
- Fertility behavior


## Home Production

- An Individual's preferences is determined by

$$
U=U(C, L)
$$

- But Goods $C$ consumed maybe purchased, in quantity $C_{M}$ or produced domestically, in quantity $C_{D}$
- Thus

$$
C=C_{D}+C_{M}
$$

## Home Production

- Total endowment of time can be divided into three parts

1. Working time in the labor market, $h_{D}$
2. Household working time, $h_{M}$
3. Leisure, $L$

- Thus

$$
T=h_{M}+h_{D}+L
$$

## Home Production

- The efficiency of household tasks is represented by a production function, thus

$$
C_{D}=f\left(h_{D}\right)
$$

- It satisfy the general characteristics of production function, thus Increasing and
Concave

$$
f^{\prime}>0, f^{\prime \prime}<0
$$

## Home Production Function

Consumption


## Market Budget Constraint

$$
C_{M} \leq w h_{M}+V
$$

- Where $C_{M}$ is consumption purchased
- $h_{M}$ denotes working hours
- $w$ is the wage rate and $V$ is still non-labor income
- We put $T=h_{M}+h_{D}+L$ in to the market budget constraint again


## Total Budget Constraint

- Then we can obtain

$$
C_{M}+w L \leq w\left(T-h_{D}\right)+V
$$

- Taking into account

$$
\begin{aligned}
C_{M} & =C-C_{D} \\
C_{D} & =f\left(h_{D}\right)
\end{aligned}
$$

- Then Total Budget Constraint

$$
C+w L \leq M+\left[f\left(h_{D}\right)-w h_{D}\right]
$$

- Where $M$ is still the potential Income, thus maximum income we can earn.


## Total Budget Constraint

$$
C+w L \leq M+\left[f\left(h_{D}\right)-w h_{D}\right]
$$

- The total income is equal to the sum of the potential income( M ) and the profit derived from household activities.
- Then, the Optimization Problem is

$$
\begin{array}{cc} 
& \operatorname{Max}_{\left\{C, L, h_{D}\right\}} U(C, L) \\
\text { s.t. } & C+w L \leq M+\left[f\left(h_{D}\right)-w h_{D}\right]
\end{array}
$$

## Optimization Solution

- The first step, we can maximize the total income of the budget constraint by choosing $h_{D}$, because home production only comes into the consumption program by this way. Thus

$$
\operatorname{Max}_{\left\{h_{D}\right\}} M+\left[f\left(h_{D}\right)-w h_{D}\right]
$$

- Then, the optimization of $h_{D}$ is defined by

$$
f^{\prime}\left(h_{D}^{*}\right)=w
$$

## Optimization Solution

- Then total potential income

$$
\tilde{M} \equiv M+f\left(h_{D}^{*}\right)-w h_{D}^{*}
$$

- So the solution

$$
\frac{U_{L}\left(C^{*}, L^{*}\right)}{U_{C}\left(C^{*}, L^{*}\right)}=w=f^{\prime}\left(h_{D}^{*}\right)
$$

- The optimal Market labor supply is

$$
T-L^{*}-h_{D}^{*} \equiv h_{M}^{*}
$$

## Optimization Solution



## Optimization Solution



## Optimization Solution



## Optimization Solution



## Home Work(Not Required)

- Analytic problems: Take into account the following situations and draw the budget lines.

1. When you go to work, you have to pay a fix cost(such as commute cost).
2. When you work for $<h$ hours(part time), wage is $w$ and when you work $>=\mathrm{h}$ (full time) hours, wage is v , and $\mathrm{v}>\mathrm{w}$.

## Family Labor Supply

## Interfamilial Decisions

- An individual who has a family makes his/her labor supply decision not individually but interfamily.
- Empirically, there are so much numerous data only describe the behavior of the household such as consumption, assets etc


## Interfamilial Decisions

- The analysis of family choices has developed along two different lines:
- The Unitary Model (Altruistic parents)
- The Collective Model(non consensus parents)


## The Unitary Model

- The traditional unitary approach to household decision making assumes
a) either family members have the same preferences
b) or individual preferences can be aggregated into a household utility function


## The Unitary Model

- A typical family with a husband and a wife, so the utility function is

$$
U=U\left(C, L_{m}, L_{f}\right)
$$

- It means that we don't care about the distribution of welfare within the household, only about the aggregate consumption level.


## The Unitary Model

- We want to maximize by choosing $\mathrm{C}, L_{m}$ and $L_{f}$

$$
\operatorname{Max}_{\left.C, L_{m}, L_{f}\right\}} U\left(C, L_{m}, L_{f}\right)
$$

- So the budget constraint is
$C+w_{m} L_{m}+w_{f} L_{f} \leq Y+\left(w_{m}+w_{f}\right) T=M$
- Where Y is the non-labor income of the household, so M is the potential income now.


## The Unitary Model

- Based on the First Order Condition(F.O.C)

$$
\begin{aligned}
& U_{C}\left(C, L_{m}, L_{f}\right)-\mu=0 \\
& U_{L m}\left(C, L_{m}, L_{f}\right)-\mu w_{m}=0 \\
& U_{L f}\left(C, L_{m}, L_{f}\right)-\mu w_{f}=0
\end{aligned}
$$

- So the optimization solution

$$
\begin{aligned}
w_{m} & =\frac{U_{L m}\left(C^{*}, L_{m}^{*} L_{f}^{*}\right)}{U_{C}\left(C^{*}, L_{m}^{*} L_{f}^{*}\right)} \\
w_{f} & =\frac{U_{L f}\left(C^{*}, L_{m}^{*} L_{f}^{*}\right)}{U_{C}\left(C^{*}, L_{m}^{*} L_{f}^{*}\right)}
\end{aligned}
$$

## The Unitary Model

- In the Unitary model, your labor supply is not only determined by your wage and your spouse's wage, as well as non-labor income.
- Now, let us combine the Home Production Model with the Unitary model to see the labor supply decisions within the family.


## Specialization and Exchange

- Based on comparative advantage theory in international trade, it will be more efficient when husband and wife specialize to product one and the other respectively.
- Then exchange their output or pool their good and income to maximize their common utility.


## Absolute Vs Comparative

TABLE 3-1 An Illustration of the Gains from Specialization and Exchange
(a) Case 1: ABSOLUTE ADVANTAGE

| Separate Production |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Value of Market Goods |  | Value of Home Cooking | Total Income |
| John | (6 hrs. $\times \$ 10$ ) |  | (2 hrs. $\times \$ 5$ ) |  |
|  | \$60 | + | \$10 | \$70 |
| Jane | (7 hrs. $\times$ \$5) |  | (1 hr. $\times \$ 10$ ) |  |
|  | \$35 | $+$ |  | \$45 |
| Total (John and Jane) | \$95 |  | \$20 | \$115 |


(b) Case 2: COMPARATIVE ADVANTAGE

| Separate Production |  |  |  |
| :--- | :--- | :---: | :---: |
|  | Value of <br> Market <br> Goods | Value of <br> Home <br> Cooking |  | | Total |
| :---: |
| Income |



## Absolute Vs Comparative

TABLE 3-1 An Illustration of the Gains from Specialization and Exchange

(b) Case 2: COMPARATIVE ADVANTAGE


|  | Specialization and Exchange |  |  |
| :---: | :---: | :---: | :---: |
|  | Value of Market Goods | Value of Home Cooking | Total Income |
| Dave | ( 8 hrs. $\times \$ 10$ ) | (0 hrs. $\times \$ 5$ ) |  |
|  | \$80 + | + \$0 | $=\$ 80$ |
| Diane | ( 6 hrs. $\times \$ 15$ ) | (2 hr. $\times \$ 15$ ) |  |
|  | \$90 + | +\$30 | $=\$ 120$ |
| Total (Dave and Diane) | \$170 | \$30 | \$200 |

## Absolute Vs Comparative

TABLE 3-1 An Illustration of the Gains from Specialization and Exchange

(b) Case 2: COMPARATIVE ADVANTAGE

| Separate Production <br>  <br>  <br> Value of <br> Market <br> GoodsValue of <br> Home <br> Cooking |  |  |  |
| :--- | :--- | :--- | :--- | | Total |
| :---: |
| Income |

(Dave
and Diane)


## Absolute Vs Comparative

TABLE 3-1 An Illustration of the Gains from Specialization and Exchange


| (b) Case 2: COMPARATIVE ADVANTAGE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Separate Production |  |  |  |  |
|  | Value of Market Goods | Value of Home Cooking |  | Total Income |
| Dave | ( 6 hrs. $\times \$ 10$ ) |  | (2 |  |
|  | \$60 | + |  | $=\$ 70$ |
| Diane | (7hrs. $\times \$ 15$ ) |  | (1 hr. $\times \$ 15$ ) |  |
|  | \$105 | + |  | $=\$ 120$ |
| Total (Dave and Diane) | \$165 |  | \$25 | \$190 |



## Absolute Vs Comparative

TABLE 3-1 An Illustration of the Gains from Specialization and Exchange


## Specialization and Exchange

- Two people: Jack and Jill
- Two goods: Market goods and home goods
- Jack is more productive at market than Jill

$$
X_{M}^{\text {Jack }}=20 L_{M}^{\text {Jack }} \quad X_{M}^{\text {Jill }}=15 L_{M}^{\text {Jill }}
$$

- Jill is more productive at home than Jack

$$
X_{H}^{\text {Jack }}=10 L_{H}^{\text {Jack }} \quad X_{H}^{\text {Jill }}=25 L_{H}^{\text {Jill }}
$$

- Both have a time constraint of 10 hours

$$
L_{M}^{\text {Jack }}+L_{H}^{\text {Jack }}=10 \quad L_{M}^{\text {Jill }}+L_{H}^{\text {Jill }}=10
$$

## Deriving Production Frontiers

- Jack:

$$
\begin{aligned}
& X_{M}^{\text {Jack }} / 20+X_{H}^{\text {Jack }} / 10=10 \\
& X_{M}^{\text {Jack }}+2 X_{H}^{\text {Jack }}=200
\end{aligned}
$$

- Jill:

$$
\begin{gathered}
X_{M}^{\text {Jill }} / 15+X_{H}^{\text {Jill }} / 25=10 \\
5 X_{M}^{\text {Jill }}+3 X_{H}^{J \text { Jill }}=750
\end{gathered}
$$

## Jack and Jill's Production Sets




## Changes in Market Wages or Home Productivity



Home productivity goes up.


Market wage goes up.

## When Jack and Jill Marry



## When Jack and Jill Marry



## When Jack and Jill Marry



## Division of Labor in the Household



Jill specializes in the household sector, Jack divides his time.


Jack specialized in the labor market, Jill divides her time.

## Division of Labor in the Household



Jack specializes in the market sector, Jill specializes in the household sector.

## Division of Labor in the Household



> When market productivity is higher than household productivity to Jill, She will do some market work and entirely housework.

## Division of Labor in the Household



When home productivity is higher than market productivity to Jack, He will still do entirely market work and some housework.

## Some Conclusions

- Specialization is caused by differences in market wages and/or home productivities, i.e., comparative advantage
- If relative market/home productivity is the same for both people, then we do not expect to see specialization.
- The model predicts that the spouse with the lower wage or with a greater home productivity tends to specialize in home production.

Dynamic Labor Supply(III)

## Overview

- Labor supply is clearly part of a lifetime decision making process.
- attend school early in life
- accumulate wealth while in labor force
- make retirement decisions late in life
- Savings from labor earnings are often required to sustain living when out of the LF.
- Variations in health, family composition and wages provide incentives for people to vary the timing of their labor earnings.


## Some Stylized Facts

- The typical age-earnings profile has a predictable path

1. low when young
2. rise as worker ages, peak at about 50
3. remain stable or decline slightly after 50

- Hours worked generally rises with age, then fall before retirement, and goes to 0 at retirement.
- Hours of work and the wage rate move together over time for a particular worker.


## FIGURE 2-18 The Life Cycle Path of Wages and Hours for a Typical Worker

(a) The age-earnings profile of a typical worker rises rapidly when the worker is young, reaches a peak at around age 50, and then wages either stop growing or decline slightly. (b) The changing price of leisure over the life cycle implies that the worker will devote relatively more hours to the labor market when the wage is high and fewer hours when the wage is low.


## A Basic Framework

- Our utility function should be in a life cycle setting.

$$
U=U\left(C_{1}, \ldots, C_{t}, \ldots, C_{T} ; L_{1}, \ldots, L_{t}, \ldots, L_{T}\right)
$$

- But we can obtain analytically simple and easily interpretable results from the function.
- So we have assume that it is temporally separable, thus

$$
U=\sum_{t=1}^{t=T} U\left(C_{t}, L_{t}, t\right)
$$

## A Basic Framework

- For a given initial value $A_{0}$, the wealth of the consumer is described by

$$
A_{t}=\left(1+r_{t}\right) A_{t-1}+B_{t}+w_{t}\left(1-L_{t}\right)-C_{t}, \quad \forall t \geq 1
$$

- It means that the wealth at any time $t$ will equal to three parts.

1. Savings and interests at $t-1$ time.
2. Non labor income at $t$ time.
3. Labor income at $t$ time.

## A Basic Framework

- The optimal solutions

$$
\begin{aligned}
\ell= & \sum_{t=1}^{t=T} U\left(C_{t}, L_{t}, t\right) \\
& -\lambda_{t} \sum_{t=}^{t=T}\left[A_{t}-\left(1+r_{t}\right) A_{t-1}-B_{t}-w_{t}\left(1-L_{t}\right)+C_{t}\right)
\end{aligned}
$$

- Then based on F.O.C, we can obtain

$$
\begin{aligned}
& U_{C}\left(C_{t}, L_{t}, t\right)=\lambda_{t} \\
& U_{L}\left(C_{t}, L_{t}, t\right)=\lambda_{t} w_{t} \\
& \lambda_{t}=\left(1+r_{t+1}\right) \lambda_{t+1}
\end{aligned}
$$

## A Basic Framework

- The equality between the MRT and the current wage is maintained at every data.
- Then the optimal consumption are implicitly written

$$
\begin{aligned}
C_{t} & =C\left(w_{t}, \lambda_{t}, t\right) \\
L_{t} & =L\left(w_{t}, \lambda_{t}, t\right)
\end{aligned}
$$

- The Frischian demands for period $t$.


## A Basic Framework

- Because $\lambda_{t}=\left(1+r_{t+1}\right) \lambda_{t+1}$
- So $\lambda_{t}=$

$$
\left[\left(1+r_{t}\right)^{-1} \times\left(1+r_{t-1}\right)^{-1} \times \ldots \times\left(1+r_{1}\right)^{-1}\right] \times
$$

$$
\lambda_{0}
$$

- Then

$$
\ln \lambda_{t}=-\sum_{i=1}^{i=t} \ln \left(1+r_{i}\right)+\ln \lambda_{0}
$$

- It can be breaks down into a fixed individual effect and an age effect common to all agents from empirical point of view.


## Two Types of Wage Changes

1. Evolutionary wage change: A wage changes along with the worker's wage-earning profile.

So It has no impact whatsoever on the worker's total lifetime income.
2. Parametric wage change: Shifts in life cycle wage profile.

## Two Types of Wage Changes

Wage Rate


Age
Evolutionary Wage Change

## Two Types of Wage Changes

Wage Rate


Evolutionary Wage Change


Parametric Wage Change

## Two Predicted Wage Effects

- Evolutionary wage change implies that hours of work and the wage rate should move together over time.
- This implication differs from the conclusion in the static model which wage increase generate income and substitute effects.
- If income effect dominate, then there could be a negative relationship between wage and working hours.


## Two Predicted Wage Effects

- Why? Because there is a huge different in "wage change" between two models.
- Static model: wage increase means that it expands the worker's opportunity set and hence creates an income effect that increase the demand of leisure.
- Lifecycle model: the wage change has no impact whatsoever on the worker's total lifetime income to a particular worker, and does not change the set.


## Parametric Wage Change

- If we compare two works with different earnings profiles, then the difference in hours of work between them would be affected by both Substitution effects and income effects.
- Labor supply may be more or less depending on which effects are stronger.


## Two Predicted Wage Effects

FIGURE 2-19 Hours of Work over the Life Cycle for Two Workers with Different Wage Paths
Joe's wage exceeds Jack's at every age. Although both Joe and Jack work more hours when the wage is high, Joe works more hours than Jack only if the substitution effect dominates. If the income effect dominates, Joe works fewer hours than Jack.


## Evolutionary Wage Change

## Wage and Labor Participation

- Labor Participation depends on the reservation wage.
- In each period of lifecycle, the worker will make decision.
- The person is then more likely to enter the labor market in periods when the wage is high.
- Then LPR are likely to be low for young workers, high for in their working years and low again for older workers.


## LFPR over Life Cycle, U.S. in 2005

FIGURE 2-20
Labor Force
Participation
Rates over the Life Cycle, 2005

Source: U.S. Bureau of Labor Statistics, Anmual Demographic Supplement of the
Current Population Surveys, 2005.


Evolutionary Wage Change

## Hour of Works Over Lifecycle,U.S. 2002

FIGURE 2-21 Hours of Work over the Life Cycle, 2005

Source: U.S. Bureau of Labor Statistics, Annual Demographic Supplement of the Current Population Surveys, 2005.


Evolutionary Wage Change

## Estimation of Life Cycle Models

- The estimation of the intertemporal labor supply elasticity.
- $\Delta \mathrm{h}_{\mathrm{it}}=\sigma \Delta \mathrm{w}_{\mathrm{it}}+$ other variables
- where $\Delta \mathrm{H}_{\mathrm{it}}$ gives the year-to-year change in hours of work.
- $\Delta \mathrm{w}_{\mathrm{it}}$ gives the year-to-year change in the worker's wages.
- The coefficient $\sigma$ would be related to the intertemporal labor supply elasticity because it measures the change in hours of work for a given person resulting from a particular change in his wage rate.


## Labor Supply Over the Business Cycle

## - Added worker effect:

- As the main breadwinner becomes unemployed, family income falls and the secondary workers get jobs to make up the loss.
- Labor force participation rate of secondary workers has a counter-cyclical trend.
- Discouraged worker effect:
- Many unemployed workers find it almost impossible to find jobs during a recession, and simply give up.
- The labor force participation rate has a pro-cyclical effect.


## Implications for Unemployment Rate

- The added worker effect increases the unemployment rate.
- The discouraged worker effect depresses the unemployment rate.
- Empirical evidence: The discourage worker effect is dominant.
- Official unemployment rate data may understate the unemployment problem during severe recession.


## Application: Retirement Decisions

- It is not linked to the deteriorating health of this particular age group.
- Partially attribute to an increase in pension benefits.
- There is a strong link between the availability of private pension plans and the labor force participation of older men.


## Social Security Disability Program

- Some studies instead argue that it can be attributed to the work disincentives created by the Social Security Disability Program.
- The monthly disability benefit equals the Social Security retirement benefits that the worker would have received had he or she continued working until age 65.
- Many workers would like to claim that they are disabled in order to enjoy the leisure activities associated with early retirement.


## Difference-in-Differences

Source: Jonathan Gruber, "Disability Insurance Benefits and Labor Supply," Journal of Political Economy 108 (December 2000): 1175.

|  | Before | After | Difference | Difference-inDifferences |
| :---: | :---: | :---: | :---: | :---: |
| Annual benefits: |  |  |  |  |
| Canada Pension Program | \$5,134 | \$7,776 | \$2,642 | \$1,666 |
| Quebec Pension Program | 6,876 | 7,852 | 976 |  |
| Percent of men aged 45 to 59 not employed last week: |  |  |  |  |
| Treatment group: CPP | 20.0\% | 21.7\% | 1.7\% | 2.7\% |
| Control group: QPP | 25.6 | 24.6 | -1.0 |  |

- The difference-in-differences estimator is $2.7 \%$.
- It implies that the increased generosity of the disability program increased the proportion of men who did not work by 2.7 percentage points.


## The Social Security Earnings Test

- Many workers who consider themselves retired continue to work, perhaps in a part-time job. In the United States, nearly $20 \%$ of "retired" persons also hold a job.
- Until 2000, the Social Security system had a provision, known as the Social Security earnings test, that presumably discouraged Social Security recipients from working.
- Annual income $<\$ 17000$, without affecting their SS benefit.
- If $>\$ 17000$, the government reduced the size of the Social Security benefit. In particular, $>\$ 1$ of Social Security benefits was withheld for every $>\$ 3$ earned above the exempt amount, so that workers who earned more than $>\$ 17,000$,implicitly faced a 33 percent tax rate.


## The Social Security Earnings Test



