Introduction to Econometrics

Recite 2 : Review of Statistical Inference

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Outlines

Population, Parameters and Random Sampling

Population, Sample and i.i.d

A **population** is a collection of people, items, or events about which you want to make inferences.

- Population always have a probability distribution.
- *A **sample*** is a subset of population, which draw from population *in a certain way*.
- To represent the population well, a sample should be randomly collected and adequately large.
  - Infinite population
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Random Sample and i.i.d

Definition

The r.v.s are called *a* ***random sample*** *of size n from the population* $f(x)$ if $X_1, ..., X_n$ are mutually independent and have the same p.d.f/p.m.f $f(x)$. Alternatively, $X_1, ..., X_n$ are called ***independent, and identically distributed*** *random variable with p.d.f/p.m.f , commonly abbreviated to i.i.d. r.v.s.*

- eg. Random sample of n respondents on a survey question.
- $X_i \perp X_j$ for all $i \neq j$
- $f_{X_i}(x)$ is the same for all i.
- And the joint p.d.f/p.m.f of $X_1, ..., X_n$ is given by

$$f(x_1, ..., x_n) = f(x_1)...f(x_n) = \prod_{i=1}^{n} f(x_i)$$

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Statistic and Sampling Distribution

Definition

$X_1, ..., X_n$ is a *random sample* of size n from the population $f(x)$. A ***statistic*** is a real-valued or vector-valued function fully depended on $X_1, ..., X_n$, thus

$$T = T(X_1, ..., X_n)$$

- and the probability distribution of a statistic T is called the ***sampling distribution*** of T.
- A statistic is only a function of the sample.

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Sample Mean and Sample Variance

Definition

The **sample average** or **sample mean**, $\overline{X}$, of the n observation $X_1, ..., X_n$ is

$$\bar{X} = \frac{1}{n}(X_1 + X_2 + ... + X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i$$

The **sample variance** is the statistic defined by

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

- if X_i is a r.v., then $\sum X_i$ is also a r.v.
- the sample mean and the sample variance are also a function of sums, so they are a r.v. too.

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A simple case of sample mean

- Let $\{X_1, X_n\} \in [1, 100]$, assume $n = 2$, thus only $X_1$and X_2

	X_1	X_2	$X_1 + X_2$	$\bar{X}$
draw 1	20	71	91	45.5
draw 2	12	66	78	39
draw 3	59	75	134	67
draw 4	3	58	61	30.5
⋮	⋮	⋮	⋮	⋮
			distribution of the sum	distribution of the mean

Statistical Inference: Estimation, Confident Intervals and Testing

Statistical Inference

- Inference
  - What is our best guess about some quantity of interest?
  - What are a set of plausible values of the quantity of interest?
- **Compare estimators, such as** in an experiment
  - we use simple difference in sample means?
  - or the post-stratification estimator, where we estimate the difference among two subsets of the data (male and female, for instance) and then take the weighted average of the two variable
  - which is better? how could we know?

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Inference: from Samples to Population

- Our focus: $\{Y_1, Y_2, ..., Y_n\}$ are i.i.d. draws from $f(y)$ or $F(Y)$, thus population distribution.
- Statistical inference or learning is using samples to infer $f(y)$.
- two ways
  - Parametric
  - Non-parametric

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Point estimation

- Point estimation: providing a single "best guess" as to the value of some fixed, unknown quantity of interest, θ, which is is a feature of the population distribution, $f(y)$.
- Examples
  - $\mu = E[Y]$
  - $\sigma^2 = Var[Y]$
  - $\mu_Y - \mu_X = E[Y] - E[X]$

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Estimator and Estimate

Definition

Given a random sample $\{Y_1, Y_2, ..., Y_n\}$ drawn from a population distribution that depends on an unknown parameter θ, and an ***estimator*** $\hat{\theta}$ is a function of the sample: thus $\hat{\theta}_n = h(Y_1, Y_2, ..., Y_n)$

- An estimator is a r.v. because it is a function of r.v.s.
  - $\{\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_n\}$ is a sequence of r.v.s, so it has convergence in probability/distribution.
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True or False and Why? "My estimate was the sample mean and my estimator was 0.5" ?

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Three Characteristics of an Estimator

- let $\hat{\mu}_Y$denote some estimator of μ_Y and $E(\hat{\mu}_Y)$ is the mean of the sampling distribution of $\hat{\mu}_Y$,

1. **Unbiasedness:** the estimator of μ_Y is ***unbiased*** if

$$E(\hat{\mu}_Y) = \mu_Y$$

2. **Consistency:**the estimator of μ_Y is ***consistent*** if

$$\hat{\mu}_Y \xrightarrow{p} \mu_Y$$

3. **Efficiency:**Let $\tilde{\mu}_Y$ be another estimator of μ_Y and suppose that both $\tilde{\mu}_Y$ and $\hat{\mu}_Y$ are unbiased.Then $\hat{\mu}_Y$ is said to be more ***efficient*** than $\hat{\mu}_Y$

$$var(\hat{\mu}_Y) < var(\tilde{\mu}_Y)$$

   - Comparing variances is difficult if we do not restrict our attention to unbiased estimators because we could always use a trivial estimator with variance zero that is biased.

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Properties of the sample mean

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$$E(\overline{Y}) = \frac{1}{n}\sum_{i=1}^{n} E(Y_i) = \mu_Y$$

so $\overline{Y}$ is an *unbiased* estimator of μ_Y.

2. Based on the L.L.N., $\overline{Y} \xrightarrow{p} \mu_Y$, so $\overline{Y}$ is also *consistent*.
3. the variance of sample mean

$$Var(\overline{Y}) = var\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(Y_i) = \frac{\sigma_Y^2}{n}$$

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$$Var(\overline{Y}) = var\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(Y_i) = \frac{\sigma_Y^2}{n}$$

4. the standard deviation of the sample mean is $\sigma_{\overline{Y}} = \frac{\sigma_Y}{\sqrt{n}}$

Properties of the sample mean

- Because efficiency entails a comparison of estimators, we need to specify the estimator or estimators to which Y is to be compared.
  - Let $\widetilde{Y} = \frac{1}{n}\left(\frac{1}{2}Y_1 + \frac{3}{2}Y_2 + \frac{1}{2}Y_3 + \frac{3}{2}Y_4 + \ldots + \frac{1}{2}Y_{n-1} + \frac{3}{2}Y_n\right)$
    - $Var(\widetilde{Y}) = 1.25\frac{\sigma_Y^2}{n} > \frac{\sigma_Y^2}{n} = Var(\overline{Y})$
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Properties of the Sample Variance

- Let μ_Y and σ_Y^2 denote the mean and variance of Y_i , then the sample variance: $S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$

1. $E(S_Y^2) = \sigma_Y^2$, thus S^2 is an *unbiased* estimator of σ_Y^2. It is also the reason why the average uses the divisor $n-1$ instead of n.
2. $S_Y^2 \xrightarrow{P} \sigma_Y^2$, thus the sample variance is a consistent estimator of the population variance.
   - Because $\sigma_{\overline{Y}} = \frac{\sigma_Y}{\sqrt{n}}$, so the statement above justifies using $\frac{S_Y}{\sqrt{n}}$ as an estimator of the standard deviation of the sample mean, $\sigma_{\overline{Y}}$.
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Interval Estimation and Confidence Intervals

The Chi-Square Distribution

- Let $Z_i(i = 1, 2, ..., m)$ be independent random variables, each distributed as **standard normal**. Then a new random variable can be defined as the sum of the squares of Z_i :

$$X = \sum_{i=1}^{m} Z_i^2$$

  Then X has a **chi-squared distribution** with m **degrees of freedom**
- The form of the distribution varies with the number of degrees of freedom, i.e. the number of standard normal random variables Z_i included in X.
- The distribution has a long tail, or is skewed, to the right. As the degrees of freedom m gets larger, however, the distribution becomes more symmetric and ``bell-shaped.'' In fact, as m gets larger, the chi-square distribution converges to, and essentially becomes, a **normal distribution**.

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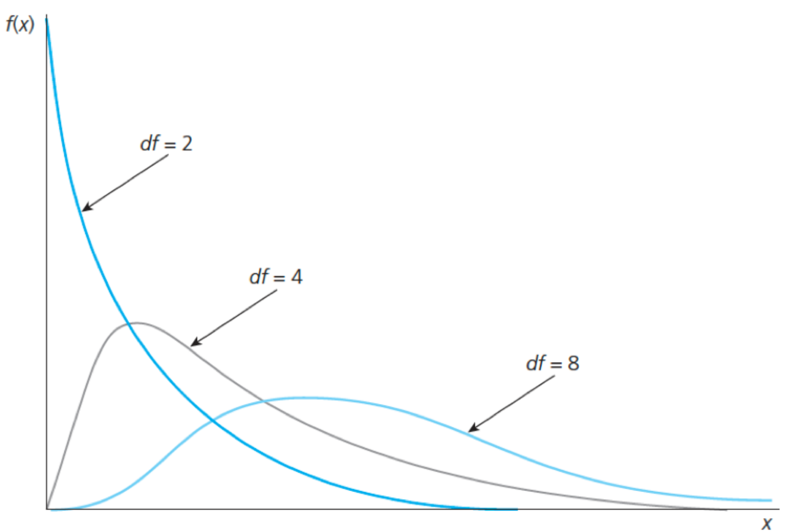
The Chi-Square Distribution

FIGURE B.9 The chi-square distribution with various degrees of freedom.



The Student t Distribution

- The Student t distribution can be obtained from a standard normal and a chi-square random variable.
- Let Z have a standard normal distribution, let X have a chi-square distribution with m degrees of freedom and assume that Z and X are independent. Then the random variable

$$T = \frac{Z}{\sqrt{X/n}}$$

  has has a t-distribution with m degrees of freedom, denoted as $T \sim t_n$.
- The shape of the t-distribution is similar to that of a normal distribution, except that the t-distribution has more probability mass in the tails.
- As the degrees of freedom get large, the t-distribution approaches **the standard normal distribution**.

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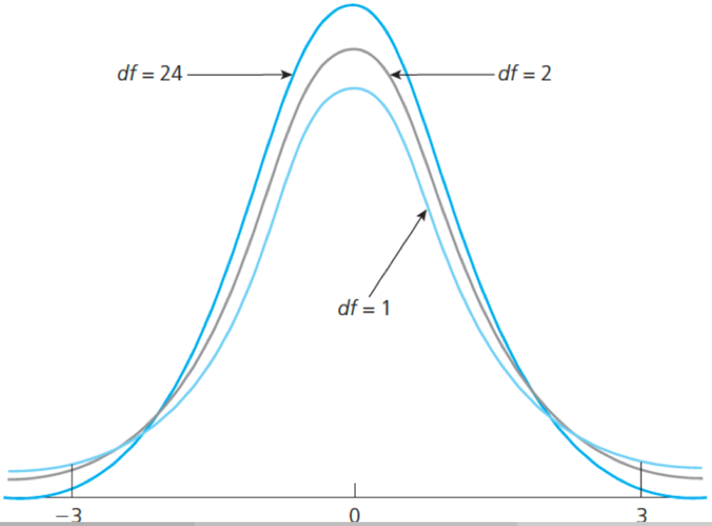
FIGURE B.10 The t distribution with various degrees of freedom.



The F Distribution

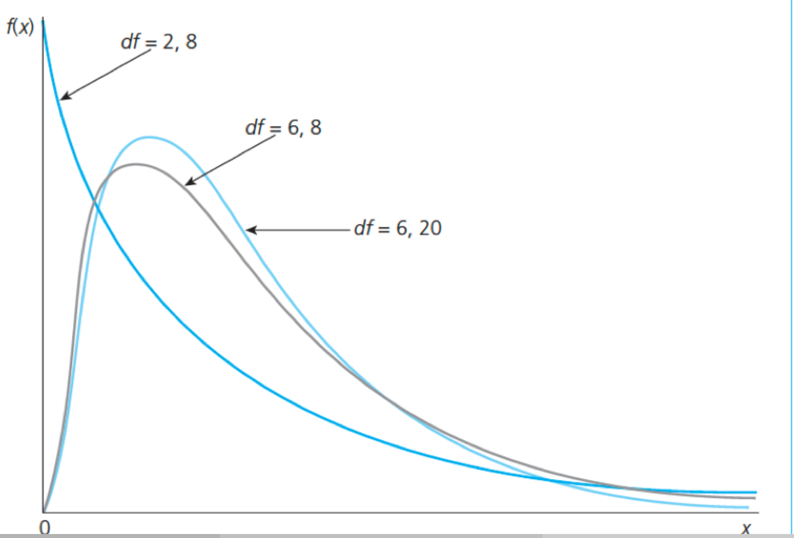
- Let $X_1 \sim \chi^2_m$and $X_2 \sim \chi^2_n$, and assume that X_1 and X_2 are independent,

$$Z = \frac{\frac{X_1}{m}}{\frac{X_2}{n}} \sim F_{m,n}$$

thus Z has an F-distribution with (m, n) degrees of freedom.

The *F* Distribution

FIGURE B.11 The F_{k_1,k_2} distribution for various degrees of freedom, k_1 and k_2.



Interval Estimation

- A point estimate provides no information about how close the estimate is "likely" to be to the population parameter.
- We cannot know how close an estimate for a particular sample is to the population parameter because the population is unknown.
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What is a Confidence Interval?

Definition

A $100(1-\alpha)\%$ confidence interval for a population parameter θ is an interval $C_n = (a, b)$, where $a = a(Y_1, ..., Y_n)$ and $b = b(Y_1, ..., Y_n)$ are functions of the data such that

$$P(a < \theta < b) = 1 - \alpha$$

- In general, this confidence level is $1-\alpha$; where α is called **significance level**.

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Interval Estimation and Confidence Intervals

- Suppose the population has a normal distribution $N(\mu, \sigma^2)$ and let $Y_1, Y_2, ..., Y_n$ be a random sample from the population.
  - Then the sample mean has a normal distribution: $\overline{Y} \sim N(\mu, \frac{\sigma^2}{n})$
  - The standardized sample mean $\overline{Z}$ is given by: $\overline{Z} = \frac{\overline{Y}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$
- Then $\theta = \overline{Z}$, then $P(a < \theta < b) = 1 - \alpha$ turns into

$$a < \frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} < b$$

then it follows that

$$P(\overline{Y} - a\sigma/\sqrt{n} < \mu < \overline{Y} + b\sigma/\sqrt{n}) = 1 - \alpha$$

- The random interval contains the population mean with a probability $1 - \alpha$.

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Interval Estimation and Confidence Intervals

- Two cases: σ is known and unknown
- When σ is known, for example,$\sigma = 1$, thus $Y \sim N(\mu, 1)$,
- then $\overline{Y} \sim N(\mu, \frac{\sigma^2}{n} = \frac{1}{n})$
- From this, we can standardize $\overline{Y}$, and, because the standardized version of $\overline{Y}$ has a standard normal distribution, and we let $\alpha = 0.05$, then we have

$$P(-1.96 < \frac{\overline{Y} - \mu}{1/\sqrt{n}} < 1.96) = 1 - 0.05$$

- The event in parentheses is identical to the event $\overline{Y} - 1.96/\sqrt{n} \leq \mu \leq \overline{Y} + 1.96/\sqrt{n}$, so

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The **t-statistic** or **t-ratio**:

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- To construct a 95% confidence interval, let c denote the 97.5^{th} percentile in the t_{n-1} distribution.

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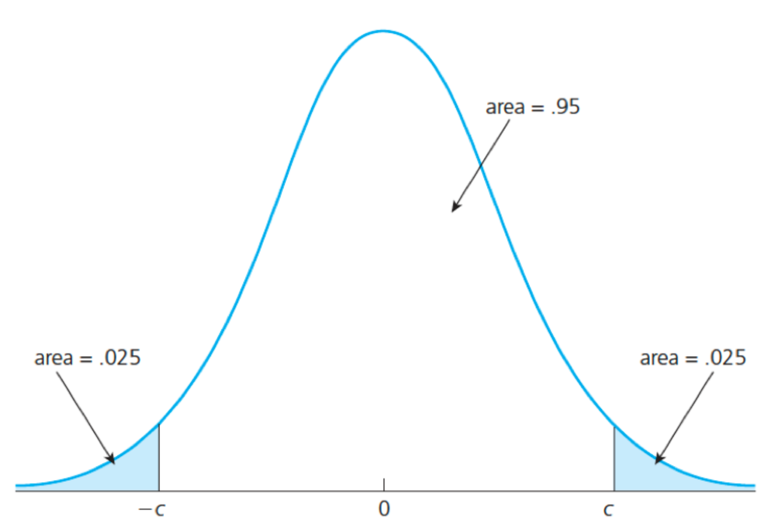
Interval Estimation and Condense Intervals

FIGURE C.4 The 97.5th percentile, c, in a t distribution.



A simple rule of thumb for a 95% confidence interval

- Caution! An often recited, but incorrect interpretation of a confidence interval is the following:
  - "I calculated a 95% confidence interval of [0.05,0.13], which means that there is a 95% chance that the true means is in that interval."
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Hypothesis Testing

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Definition

A hypothesis is a statement about a population parameter, thus θ. Formally, we want to test whether is significantly different from a certain value μ_0

$$H_0 : \theta = \mu_0$$

which is called **null hypothesis**. **The alternative hypothesis** is

$$H_1 : \theta \neq \mu_0$$

- If the value μ_0 does not lie within the calculated condense interval, then we **reject** the null hypothesis.
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General framework

- A hypothesis test chooses whether or not to reject the null hypothesis based on the data we observe.
- Rejection based on a test statistic

$$T_n = T(Y_1, ..., Y_n)$$

- The null/reference distribution is the distribution of T under the null.
- We'll write its probabilities as $P_0(T_n \leq t)$

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Two Type Errors

- In both cases, there is a certain risk that our conclusion is wrong

Type I Error

A Type I error is when we reject the null hypothesis when it is in fact true.("left-wing")

- We say that the Lady is discerning when she is just guessing(null hypo: she is just guessing)

Type II Error

A Type II error is when we fail to reject the null hypothesis when it is false.("right-wing")

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P-Value

- To provide additional information, we could ask the question: What is the largest significance level at which we could carry out the test and still fail to reject the null hypothesis?
- We can consider the **p-value** of a test
  1. Calculate the t-statistic t
  2. The largest significance level at which we would fail to reject H_0 is the significance level associated with using t as our critical value

$$p - value = 1 - \Phi(t)$$

where denotes the standard normal c.d.f.(we assume that n is large enough)

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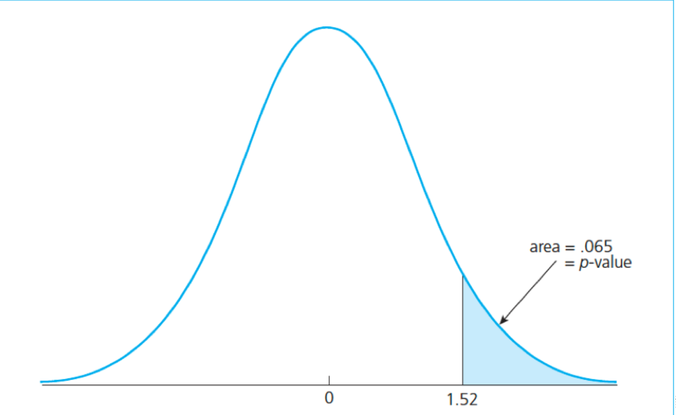
P-Value

- Suppose that $t = 1.52$, then we can find the largest significance level at which we would fail to reject H_0

$$p - value = P(T > 1.52 \mid H_0) = 1 - \Phi(1.52) = 0.065$$

FIGURE C.7 The p-value when $t = 1.52$ for the one-sided alternative μ . μ_0.



Comparing Means from Different Populations

An Example: Comparing Means from Different Populations

- Do recent male and female college graduates earn the same amount on average? This question involves comparing the means of two different population distributions.
- In an RCT, we would like to estimate the average causal effects over the population

$$ATE = ATT = E\{Y_i(1) - Y_i(0)\}$$

- We only have random samples and random assignment to treatment, then what we can estimate instead

$$\textit{difference in mean} = \overline{Y}_{treated} - \overline{Y}_{control}$$

- Under randomization, *difference-in-means* is a good estimate for the ATE.

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Hypothesis Tests for the Difference Between Two Means

- To illustrate a test for the difference between two means, let mw be the mean hourly earning in the population of women recently graduated from college and let mm be the population mean for recently graduated men.
- Then the **null hypothesis** and **the two-sided alternative hypothesis** are

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\begin{aligned}
H_0 : \mu_m &= \mu_w \\
H_1 : \mu_m &\neq \mu_w
\end{aligned}
$$

- Consider the null hypothesis that mean earnings for these two populations differ by a certain amount, say d_0. The null hypothesis that men and women in these populations have the same mean earnings corresponds to H_0 : $H_0 : d_0 = \mu_m - \mu_w = 0$

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- Let us discuss the distribution of $\overline{Y}_m - \overline{Y}_w$.

$$\sim N(\mu_m - \mu_w, \frac{\sigma_m^2}{n_m} + \frac{\sigma_w^2}{n_w})$$

- if $\sigma_m^2$and σ_w^2 are known, then the this approximate normal distribution can be used to compute p-values for the test of the null hypothesis. In practice, however, these population variances are typically unknown so they must be estimated.
- Thus the *standard error* of $\overline{Y}_m - \overline{Y}_w$ is

$$SE(\overline{Y}_m - \overline{Y}_w) = \sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}$$

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$$t = \frac{\overline{Y}_m - \overline{Y}_w - d_0}{SE(\overline{Y}_m - \overline{Y}_w)}$$

- If both $n_m$and n_m are large, then this t-statistic has a standard normal distribution when the null hypothesis is true.

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Confidence Intervals for the Difference Between Two Population Means

- the 95% two-sided confidence interval for d consists of those values of d within ± 1.96 standard errors of $\overline{Y}_m - \overline{Y}_w$, thus $d = \mu_m - \mu_w$ is

$$(\overline{Y}_m - \overline{Y}_w) \pm 1.96 SE(\overline{Y}_m - \overline{Y}_w)$$

Wrap Up