Lecture 7A: Fixed Effect Model

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Section 1

Panel Data: What and Why

Introduction

- So far, we have only focused on data cross entities.Now it is the time to add time, which leads us to use **Panel Data**.
- A panel dataset contains observations on multiple entities, where each entity is observed at two or more points in time.
- If the data set contains observations on the variables X and Y,then the data are denoted

$$(X_{it}, Y_{it}),\ i = 1, ...n\ and\ t = 1, ..., T$$

  - the first subscript,i refers to the entity being observed
  - the second subscript,t refers to the date at which it is observed

- Whether some observations are missing
  - **balanced** panel
  - **unbalanced** panel

Introduction: Data Structure

TABLE 1.3 Selected Observations on Cigarette Sales, Prices, and Taxes, by State and Year for U.S. States, 1985–1995

Observation Number	State	Year	Cigarette Sales (packs per capita)	Average Price per Pack (including taxes)	Total Taxes (cigarette excise tax + sales tax)
1	Alabama	1985	116.5	$1.022	$0.333
2	Arkansas	1985	128.5	1.015	0.370
3	Arizona	1985	104.5	1.086	0.362
⋮	⋮	⋮	⋮	⋮	⋮
47	West Virginia	1985	112.8	1.089	0.382
48	Wyoming	1985	129.4	0.935	0.240
49	Alabama	1986	117.2	1.080	0.334
⋮	⋮	⋮	⋮	⋮	⋮
96	Wyoming	1986	127.8	1.007	0.240
97	Alabama	1987	115.8	1.135	0.335
⋮	⋮	⋮	⋮	⋮	⋮

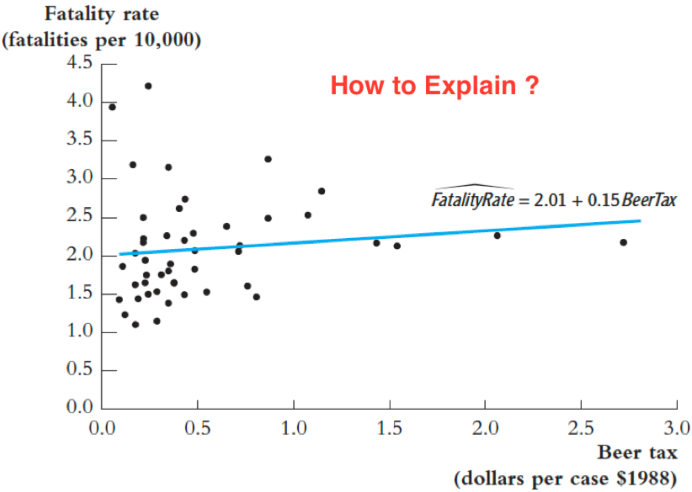
Example: Traffic deaths and alcohol taxes

- Observational unit: *one* year in *one* U.S. state
- 48 U.S. states, so n = the number of entities = 48 and 7 years (1982,..., 1988),so T = # of time periods = 7. Balanced panel, so total number of observations $= 7 \times 48 = 336$
- Variables:
  - Dependent Variable: **Traffic fatality rate** (# traffic deaths in that state in that year, per 10,000 state residents)
  - Independent Variable: **Tax on a case of beer**
  - Other Controls (legal driving age, drunk driving laws, etc.)
- A simple OLS regression model with $t = 1982, 1988$

$$FatalityRate_{it} = \beta_{0t} + \beta_{1t} BeerTax_{it} + u_{it}$$

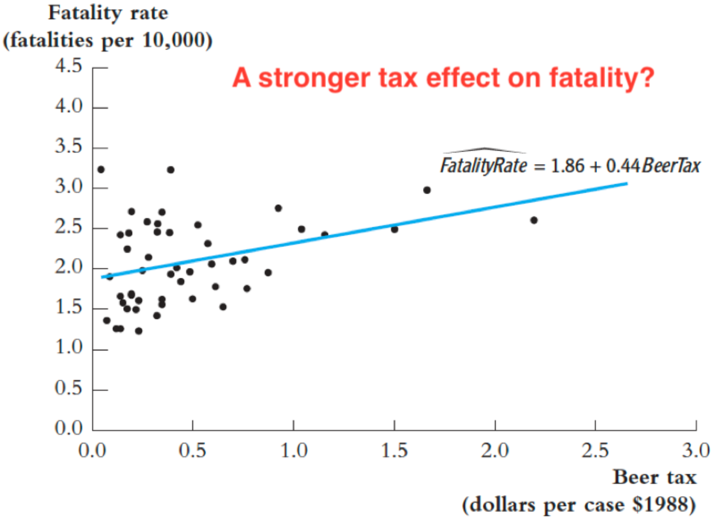
U.S. traffic death data for 1982

- Higher alcohol taxes, more traffic deaths



U.S. traffic death data for 1988

- Still higher alcohol taxes, more traffic deaths



Simple Case: Panel Data with Two Time Periods

- Let adjust our model with some unobservables

$$FatalityRate_{it} = \beta_0 + \beta_1 BeerTax_{it} + \beta_2 Z_i + u_{it}$$

where u_{it} is the error term and $i = 1, ...n \ and \ t = 1, ..., T$

- Z_i is the unobservable factor that determines the fatality rate in the i state but **does not change over time**.
  - eg. local cultural attitude toward drinking and driving.
- The omission of Z_i might cause omitted variable bias but we don't have data on Z_i.
- The key idea: *Any change* in the fatality rate from 1982 to 1988 cannot be caused by Z_i, because Z_i (by assumption) *does not change* between 1982 and 1988.

Panel Data with Two Time Periods:Before and After Model

- Consider the regressions for 1982 and 1988...

$$FatalityRate_{i1988} = \beta_0 + \beta_1 BeerTax_{i1988} + \beta_2 Z_i + u_{i1988}$$
$$FatalityRate_{i1982} = \beta_0 + \beta_1 BeerTax_{i1982} + \beta_2 Z_i + u_{i1982}$$

- Then make a difference

$$FatalityRate_{i1988} - FatalityRate_{i1982} =$$
$$\beta_1(BeerTax_{i1988} - BeerTax_{i1982}) + (u_{i1988} - u_{i1982})$$

Panel Data with Two Time Periods

- Assumption: if $E(u_{it}|BeerTax_{it}, Z_{it}) = 0$,then $(u_{i1988} - u_{i1982})$ is uncorrelated with $(BeerTax_{i1988} - BeerTax_{i1982})$
- Then this "difference" equation can be estimated by OLS, even though Z_i isn't observed.
- Because the omitted variable Z_i doesn't change, it cannot be a determinant of the change in Y.

Case: Traffic deaths and beer taxes

1982 data:

$$\widehat{FatalityRate} = \underset{(.11)}{1.86} + \underset{(.13)}{0.44}BeerTax \qquad (n = 48)$$

1988 data:

$$\widehat{FatalityRate} = \underset{(.15)}{2.01} + \underset{(.13)}{0.15}BeerTax \qquad (n = 48)$$

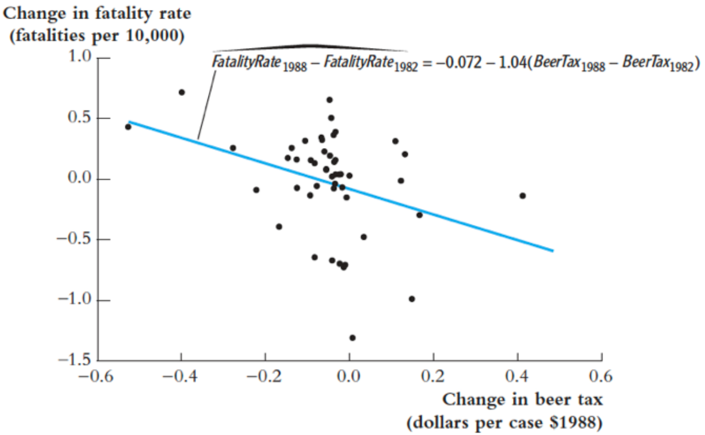
Difference regression $(n = 48)$

$$\widehat{FR_{1988} - FR_{1982}} = \underset{(.065)}{-.072} - \underset{(.36)}{1.04}(BeerTax_{1988} - BeerTax_{1982})$$

Change in traffic deaths and change in beer taxes

FIGURE 10.2 Changes in Fatality Rates and Beer Taxes, 1982–1988

This is a scatterplot of the *change* in the traffic fatality rate and the *change* in real beer taxes between 1982 and 1988 for 48 states. There is a negative relationship between changes in the fatality rate and changes in the beer tax.



Wrap up

- In contrast to the cross-sectional regression results, the estimated effect of a change in the real beer tax is **negative**, as predicted by economic theory.
- By examining changes in the fatality rate over time, the regression *controls for fixed factors* such as cultural attitudes toward drinking and driving.
- But there are many factors that influence traffic safety, and if they change over time and are correlated with the real beer tax, then their omission will produce omitted variable bias.

Wrap up

- This "before and after" analysis works *when the data are observed in **two** different years*.
- Our data set, however, contains observations for **seven** different years,and it seems foolish to discard those potentially useful additional data.
- But the "before and after" method does not apply directly when $T > 2$. To analyze all the observations in our panel data set, we use a more general regression setting: **fixed effects**

Section 2

Fixed Effects Model

Introduction

- Fixed effects regression is a method for controlling for omitted variables in panel data when *the omitted variables vary across entities (states) but do not change over time*.
- Unlike the "before and after" comparisons,fixed effects regression can be used when there are **two or more time** observations for each entity.

Fixed Effects Regression Model

- The **dependent variable** (FatalityRate) and **indenpendent variable** (BeerTax) denoted as Y_{it} and X_{it}, respectively. Then our model is

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 Z_i + u_{it} \tag{7.1}$$

- Where Z_i is an **unobserved variable** that varies from one state to the next but **does not change over time**
  - eg. Z_i can still represent cultural attitudes toward drinking and driving.
- We want to estimate β_1, the effect on Y of X holding constant the unobserved state characteristics Z.

Fixed Effects Regression Model

- Because Z_i varies from one state to the next but is constant over time,then let $\alpha_i = \beta_0 + \beta_2 Z_i$,the Equation becomes

$$Y_{it} = \beta_1 X_{it} + \alpha_i + u_{it} \tag{7.2}$$

- This is the **fixed effects regression model**, in which α_i are treated as *unknown intercepts* to be estimated, one for each state. The interpretation of α_i as a *state-specific intercept* in Equation (7.2).
- Because the intercept α_i can be thought of as the "effect" of being in entity i (in the current application, entities are states, the terms α_i,known as **entity fixed effects**.
- The variation in the entity fixed effects comes from omitted variables that, like Z_i in Equation (7.1), vary across entities but not over time.

Alternative : Fixed Effects by using binary variables

- How to estimate these parameters α_i.
- To develop the fixed effects regression model using binary variables, let $D1_i$ be a binary variable that equals 1 when i = 1 and equals 0 otherwise, let $D2_i$ equal 1 when i = 2 and equal 0 otherwise, and so on.
- Arbitrarily omit the binary variable $D1_i$ for the first group. Accordingly, the fixed effects regression model in Equation (7.2) can be written equivalently as

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \gamma_2 D2_i + \gamma_3 D3_i + ... + \gamma_n Dn_i + u_{it} \quad (7.3)$$

- Thus there are two equivalent ways to write the fixed effects regression model, Equations (7.2) and (7.3).
- In both formulations, the slope coefficient on X is the same from one state to the next.

Estimation and Inference

- In principle the binary variable specification of the fixed effects regression model can be estimated by OLS.
- But it is tedious to estimate so many fixed effects.If $n = 1000$, then you have to estimate $1000 - 1 = 999$ fixed effects.
- There are some special routines, which are equivalent to using OLS on the full binary variable regression, are *faster* because they employ some *mathematical simplifications* that arise in the algebra of fixed effects regression.

Estimation: The "entity-demeaned"

- Computes the OLS fixed effects estimator in two steps
- The **first** step:
  - take the average across times t of both sides of Equation (7.2);

$$\bar{Y}_i = \beta_1 \bar{X}_i + \alpha_i + \bar{u}_t \tag{7.4}$$

  - demeaned: let Equation(7.2) minus (7.4)

$$Y_{it} - \bar{Y}_i = \beta_1 X_{it} - \bar{X}_i + (\alpha_i - \alpha_i) + u_{it} - \bar{u}_i$$

Estimation: The "entity-demeaned"

- Let

$$\tilde{Y}_{it} = Y_{it} - \bar{Y}_i$$
$$\tilde{X}_{it} = X_{it} - \bar{X}_i$$
$$\tilde{u}_{it} = u_{it} - \bar{u}_i$$

- Then the **second** step: accordingly,estimate

$$\tilde{Y}_{it} = \beta_1 \tilde{X}_{it} + \tilde{u}_{it} \quad (7.5)$$

- Then the estimator is known as the **within estimator**. Because it matters not if a unit has consistently high or low values of Y and X. All that matters is how the variations around those mean values are correlated.
- In fact, this estimator is identical to the OLS estimator of β_1 without intercept obtained by estimation of the fixed effects model in Equation (7.3)

OLS estimator without intercept

- OLS estimator without intercept

$$Y_i = \beta_1 X_i + u_i$$

- The least squared term

$$\min_{b_1} \sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (Y_i - b_1 X_i)^2$$

- F.O.C, thus differentiating with respect to β_1, we get

$$\sum_{i=1}^{n} 2(Y_i - b_1 X_i) X_i = 0$$

- At last,

$$\hat{\beta}_1 = b_1 = \frac{\sum_{i=1}^{n} Y_i X_i}{\sum_{i=1}^{n} X_i^2}$$

Fixed effects estimator(I)

- The second step:

$$\tilde{Y}_{it} = \beta_1 \tilde{X}_{it} + \tilde{u}_{it} \tag{7.4}$$

- Then the fixed effects estimator can be obtained based on OLS estimator without intercept

$$\hat{\beta}_{fe} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T} \tilde{Y}_{it} \tilde{X}_{it}}{\sum_{i=1}^{n} \sum_{t=1}^{T} \tilde{X}_{it}^2}$$

Fixed effect estimator(II)

- The fixed effects model is

$$Y_{it} = \beta_1 X_{it} + \alpha_i + u_{it} \tag{7.2}$$

- Equivalence to

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \gamma_2 D2_i + \gamma_3 D3_i + ... + \gamma_n Dn_i + u_{it} \tag{7.3}$$

- Then we can think of α_i as fixed effects or "nuisance parameters" to be estimated,thus yields

$$(\hat{\beta}, \hat{\alpha}_1, ..., \hat{\alpha}_n) = \underset{b, a_1, ..., a_n}{\operatorname{argmin}} \sum_{i=1}^{n} \sum_{t=1}^{T} (Y_{it} - bX_{it} - a_i)^2$$

this amounts to including $n = n + 1 - 1$ dummies in regression of Y_{it} on X_{it}

Fixed effect estimator(II)

- The first-order conditions (FOC) for this minimization problem are:

$$\sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it}-\hat{\beta}X_{it}-\hat{\alpha}_i)X_{it}=0$$

- And

$$\sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it}-\hat{\beta}X_{it}-\hat{\alpha}_i)=0$$

Fixed effect estimator(II)

- Therefore, for $i = 1, \dots, N$,

$$\hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}(Y_{it} - \hat{\beta}X_{it}) = \overline{Y}_i - \overline{X}_i\hat{\beta},$$

where

$$\bar{X}_i \equiv \frac{1}{T}\sum_{t=1}^{T}X_{it}; \bar{Y}_i \equiv \frac{1}{T}\sum_{t=1}^{T}Y_{it}$$

Fixed effect estimator(II)

- Plug this result into the first FOC to obtain:

$$
\begin{aligned}
\sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it}-\hat{\beta}X_{it}-\hat{\alpha}_i)X_{it} &= \sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it}-X_{it}\hat{\beta}-\overline{Y}_i+\overline{X}_i\hat{\beta})X_{it} \\
&= \left(\sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it}-\overline{Y}_i)X_{it}\right) \\
&\quad -\hat{\beta}\left(\sum_{i=1}^{n}\sum_{t=1}^{T}(X_{it}-\bar{X}_i)X_{it}\right)=0
\end{aligned}
$$

Fixed effect estimator(II)

- Then we could obtain

$$\hat{\beta} = \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}(X_{it}-\bar{X}_i)(X_{it}-\bar{X}_i)}{\sum_{i=1}^{n}\sum_{t=1}^{T}(Y_{it}-\bar{Y})(X_{it}-\bar{X}_i)} = \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}\tilde{Y}_{it}}{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}^2}$$

  with time-demeaned variables $\tilde{X}_{it} \equiv X_{it} - \bar{X}, \tilde{Y}_{it} \equiv Y_{it} - \bar{Y}_i$

- which is same as we obtained in demeaned method.

The Fixed Effects Regression Assumptions

- The simple fixed effect model

$$Y_{it} = \beta_1 X_{it} + \alpha_i + u_{it}, i = 1, ...n \ t = 1, ..., T$$

1. Assumption 1: u_{it} has conditional mean zero with X_it, or X_i at any time and α_i
$$E(u_{it}|X_{i1}, X_{i2}, ..., X_{iT}, \alpha_i) = 0$$
2. Assumption 2: $(X_{i1}, X_{i2}, ..., X_{iT}, u_{i1}, u_{i2}, ..., u_{iT}), i = 1, 2, ..., n$ are $i.i.d$
3. Assumption 3: Large outliers are unlikely.
4. Assumption 4: There is no perfect multicollinearity.

- For multiple regressors, X_{it} should be replaced by the full list $X_{1,it}, X_{2,it}, ..., X_{k,it}$

Statistical Properties

- Unbiasedness and Consistency

$$
\begin{aligned}
\hat{\beta}_{fe} &= \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}\tilde{Y}_{it}}{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}^2} \\
&= \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}(\beta_1\tilde{X}_{it}+\tilde{u}_{it})}{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}^2} \\
&= \beta_1 + \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}\tilde{u}_{it}}{\sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{X}_{it}^2}
\end{aligned}
$$

- It is very familiar: paralleling the derivation of OLS estimator, we could prove the estimator of fixed effects model is **unbiased** and **consistent**.

Statistical Properties

- Similarly, in panel data, if the fixed effects regression assumptions—holds, then the sampling distribution of the fixed effects OLS estimator is normal in large samples,
- Then the variance of that distribution can be estimated from the data, the square root of that estimator is the standard error,
- And the standard error can be used to construct t-statistics and confidence intervals.
- Statistical inference—testing hypotheses (including joint hypotheses using F-statistics) and constructing confidence intervals—proceeds in exactly the same way as in multiple regression with cross-sectional data.

Fixed Effects: Extension to multiple X's.

- The fixed effects regression model is

$$Y_{it} = \beta_1 X_{1,it} + ... + \beta_k X_{k,it} + \alpha_i + u_{it}$$

- Equivalently, the fixed effects regression can be expressed in terms of a common intercept

$$\begin{aligned} Y_{it} =& \beta_0 + \beta_1 X_{1,it} + ... + \beta_k X_{k,it} \\ &+ \gamma_2 D2_i + \gamma_3 D3_i + ... + \gamma_n Dn_i + u_{it} \end{aligned}$$

Application to Traffic Deaths

- The OLS estimate of the fixed effects regression based on all 7 years of data (336 observations), is

$$\widehat{FatalityRate} = -0.66BeerTax + StateFixedEffects$$
$$(0.29)$$

- The estimated state fixed intercepts are not listed to save space and because they are not of primary interest.
- As predicted by economic theory,higher real beer taxes are associated with fewer traffic deaths, which is the opposite of what we found in the initial cross-sectional regressions.

Application to Traffic Deaths

- Recall: The result in Before-After Model is

Difference regression ($n = 48$)

$$\widehat{FR_{1988} - FR_{1982}} = -.072 - 1.04(BeerTax_{1988} - BeerTax_{1982})$$
$$(.065) \quad (.36)$$

- The magnitudes of estimate coefficients are not identical,because they use different data.
- And because of the additional observations, the standard error now is also smaller than before-after model.

Section 3

Extension: Both Entity and Time Fixed Effects

Regression with Time Fixed Effects

- Just as fixed effects for each entity can control for variables that are constant over time but differ across entities, so can **time fixed effects** control for variables that *are constant across entities but evolve over time*.
  - Like *safety improvements in new cars* as an **omitted variable** that changes over time but has the same value for all states.
- Now our regression model with **time fixed effects**

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_3 S_t + u_{it}$$

- where S_t is *unobserved* and where the single t subscript emphasizes that safety changes over time but is constant across states. Because $\beta_3 S_3$ represents variables that determine Y_{it}, if S_t is correlated with X_{it}, then omitting S_t from the regression leads to omitted variable bias.

Time Effects Only

- Although S_t is unobserved, its influence can be eliminated because it varies over time but not across states, just as it is possible to eliminate the effect of Z_i, which varies across states but not over time.
- Similarly,the presence of S_t leads to a regression model in which each time period has its own intercept,thus

$$Y_{it} = \beta_1 X_{it} + \lambda_t + u_{it}$$

- This model has a different intercept, λ_t, for each time period, which are known as **time fixed effects**.The variation in the time fixed effects comes from omitted variables that vary over time but not across entities.

Time Effects Only

- Just as the **entity fixed effects** regression model can be represented using $n-1$ binary indicators, the time fixed effects regression model be represented using $T-1$ binary indicators too:

$$Y_{it} = \beta_0 + \beta_1 X_{1,it} + \delta_2 B2_t + ... + \delta_T BT_t + \alpha_i + u_{it} \quad (10.18)$$

  - where $\delta_2, \delta_3, ..., \delta_T$ are unknown coefficients
  - where $B2_t = 1$ if $t = 2$ and $B2_t = 0$ otherwise and so forth.

- Nothing new, just a another form of Fixed Effects model with another explanation.

Both Entity and Time Fixed Effects

- If some omitted variables are constant over time but vary across states (such as cultural norms) while others are constant across states but vary over time (such as national safety standards)
- Then, combined entity and time fixed effects regression model is

$$Y_{it} = \beta_1 X_{it} + \alpha_i + \lambda_t + u_{it}$$

  - where α_i is the **entity fixed effect** and λ_t is the **time fixed effect**.

- This model can equivalently be represented as follows

$$\begin{aligned} Y_{it} =& \beta_0 + \beta_1 X_{it} + \gamma_2 D2_i + \gamma_3 D3_i + ... + \gamma_n Dn_i \\ &+ \delta_2 B2_t + \delta_3 B3_t + ... + \delta_T BT_i + u_{it} \end{aligned}$$

Both Entity and Time Fixed Effects: Estimation

- The time fixed effects model and the entity and time fixed effects model are both **variants** of *the multiple regression model*.
- Thus their coefficients can be estimated by OLS by including the additional time and entity binary variables.
- Alternatively,first deviating Y and the X's from their entity and time-period means and then by estimating the multiple regression equation of deviated Y on the deviated X's.

Application to traffic deaths

- This specification includes the beer tax, 47 state binary variables (state fixed effects), 6 single-year binary variables (time fixed effects), and an intercept, so this regression actually has $1 + 47 + 6 + 1 = 55$ right-hand variables!

$$\widehat{FatalityRate} = \underset{(0.36)}{-0.64} BeerTax + StateFixedEffects + TimeFixedEffects. \quad (10.21)$$

- When time effects are included,this coefficient is less precisely estimated, it is still significant only at the 10%, but not the 5%.
- This estimated relationship between the real beer tax and traffic fatalities is immune to omitted variable bias from variables that are constant either over time or across states.

Autocorrelated in Panel Data

- An important difference between the panel data assumptions in Key Concept 10.3 and the assumptions for cross-sectional data in Key Concept 6.4 is Assumption 2.
  - **Cross-Section**: Assumption 2 holds: i.i.d sample.
  - **Panel data**: independent across entities but no such restricition **within** an entity.
- Like X_{it} can be correlated over time within an entity, thus $Cov(X_t, X_s)$ for some $t \neq s$, then the X_t is said to be **autocorrelated or serially correlated**.

Autocorrelated in Panel Data

- In the traffic fatality example, X_{it}, the beer tax in state i in year t,is autocorrelated:
  - Most of the time, the legislature does not change the beer tax, so if it is high one year relative to its mean value for state i,it will tend to be high the next year,too.

Autocorrelated in Panel Data

- Similarly, u_{it} would be also autocorrelated. It consists of time-varying factors that are determinants of Y_{it} but are not included as regressors, and some of these omitted factors might be autocorrelated. It can formally be expressed as

$$Cov(u_{it}, u_{is}|X_{it}, X_{is}, \alpha_i) \neq 0 \, for \, t \neq s$$

  - eg. a downturn in the local economy and a road improvement project.

Autocorrelated in Panel Data

- If the regression errors are autocorrelated, then the usual heteroskedasticity-robust standard error formula for cross-section regression is not valid.
- The result: an analogy of heteroskedasticity.
- OLS panel data estimators of β are unbiased and consistent but the standard errors will be wrong
  - usually the OLS standard errors understate the true uncertainty
- This problem can be solved by using **"heteroskedasticity and autocorrelation-consistent(HAC) standard errors"**

Standard Errors for Fixed Effects Regression

- The standard errors used are one type of HAC standard errors, **clustered standard errors**.
- The term **clustered** arises because these standard errors allow the regression errors to have an arbitrary correlation within a cluster, or grouping, but assume that the regression errors are uncorrelated across clusters.
- In the context of panel data,each cluster consists of an entity.Thus **clustered standard errors** allow for heteroskedasticity and for arbitrary autocorrelation *within an entity*, but treat the errors as *uncorrelated across entities*.
- Like **heteroskedasticity-robust standard errors** in regression with cross-sectional data, **clustered standard errors** are valid whether or not there is heteroskedasticity,autocorrelation,or both.

Application: Drunk Driving Laws and Traffic Deaths

- Two ways to cracks down on Drunk Driving
  1. toughening driving laws
  2. raising taxes
- Both driving laws and economic conditions could be omitted variables

Application: Drunk Driving Laws and Traffic Deaths

TABLE 10.1 Regression Analysis of the Effect of Drunk Driving Laws on Traffic Deaths

Dependent variable: Traffic fatality rate (deaths per 10,000).

Regressor	(1) OLS	(2) Only State Fixed	(3)	(4) Both State and Time Fixed Effects	(5)	(6)	(7)
Beer tax	0.36** (0.05)	−0.66* (0.29)	−0.64+ (0.36)	−0.45 (0.30)	−0.69* (0.35)	−0.46 (0.31)	−0.93** (0.34)
Drinking age 18				0.028 (0.070)	−0.010 (0.083)		0.037 (0.102)
Drinking age 19				−0.018 (0.050)	−0.076 (0.068)		−0.065 (0.099)
Drinking age 20				0.032 (0.051)	−0.100+ (0.056)		−0.113 (0.125)
Drinking age						−0.002 (0.021)	
Mandatory jail or community service?				0.038 (0.103)	0.085 (0.112)	0.039 (0.103)	0.089 (0.164)
Average vehicle miles per driver				0.008 (0.007)	0.017 (0.011)	0.009 (0.007)	0.124 (0.049)
Unemployment rate				−0.063** (0.013)		−0.063** (0.013)	−0.091** (0.021)
Real income per capita (logarithm)				1.82** (0.64)		1.79** (0.64)	1.00 (0.68)
Years	1982–88	1982–88	1982–88	1982–88	1982–88	1982–88	1982 & 1988 only
State effects?	no	yes	yes	yes	yes	yes	yes
Time effects?	no	no	yes	yes	yes	yes	yes
Clustered standard errors?	no	yes	yes	yes	yes	yes	yes

Wrap up

- We've showed that how panel data can be used to control for unobserved omitted variables that differ across entities but are constant over time.
- The key insight is that if the unobserved variable does not change over time, then any changes in the dependent variable must be due to influences other than these fixed characteristics.
- Double fixed Effects model, thus both entity and time fixed effects can be included in the regression to control for variables that vary across entities but are constant over time and for variables that vary over time but are constant across entities.
- Despite these virtues, entity and time fixed effects regression *cannot* control for *omitted variables* that *vary both across entities and over time*.There remains a need for a new method that can eliminate the influence of unobserved omitted variables.